A public exhibition on Mathematics of Planet Earth

Some potential examples of modules on four themes

A planet to discover

1. Crystallography: the crystallographic groups describe the different possible symmetries of the tilings of 3-dimensional space which are invariant under three independent translations. There are applications in the structure of crystals inside rocks. More generally, in chemistry, crystallography is the science of the arrangements of atoms inside a solid. Some arrangements are denser than others and the density of the packings is related to the chemical properties of the chemical elements. On the mathematical side, density of packings is linked with Kepler conjecture on the densest packing of spheres. Different densities can be studied: the densest one, the random density when spheres are packed at random. The same questions can be asked for objects with other shapes than spheres. More recently, mineralogical finding offered evidence that quasicrystals might form naturally under suitable geological conditions.

2. Fractals provide models for the shapes of nature: rocky coasts, ferns, the networks of brooks and rivers, for instance deltas. The fractal dimension is a measure of the “density” of a fractal which allows to compare the density of different fractals

3. The movements of the Earth and the planets in the Solar system: the inner planets (Mercury, Venus, the Earth and Mars) have chaotic motions. Simulations show a 1% chance that Mercury be destabilized and encounters a collision with the Sun or Venus. There is a much smaller chance that all the inner planets be destabilized and that there could be a collision between the Earth and either Venus or Mars in ~3.3Gyr (Jacques Laskar, 2009)

4. The role of the Moon to stabilize the axis of the Earth. If we remove the Moon, then simulations show that the Earth’s axis would undergo large oscillations and we would not experience the climates that we now have. In the same spirit there are recent studies of Jacques Laskar making the link between the changes of the parameters of the Earth: angle of the axis, excentricity of the orbit, etc. and the past climates of the Earth (glaciation periods)

5. Why the seasons? This theme is very standard. But, in many countries, it disappears from basic science education and needs to be taught independently.

6. The eclipses. Two types of eclipses: Sun eclipses of Moon eclipses. Explanation of the phenomenon. Previsions of the eclipses.

7. Weather previsions. The use of models. The butterfly effect in meteorology.

8. Remote sensing for exploring the Earth. It could be the use of aerial photographs to discover resources or the use of seismic waves do discover resources in the underground.

9. Localizing events: earthquakes, thunderstorms, etc. This is done through triangulation when several distant stations note the time when they register the event.

10. The Global Positioning System (GPS): how it works. Applications to measuring the height of mountains like Mount Everest and Mont Blanc and evaluating their growth.

11. Elements of cartography. It is not possible to draw a map of the Earth respecting ratios of distances. All maps need to make compromise. The Lambert equivalent projection preserves ratios of areas. The Mercator projection preserves angles. Other types of projections have also remarkable properties.

12. Drawing maps of the Earth with its chains of mountains that give the illusion of altitude using stereoscopy (3-D imaging). How to produce the map? How to look at it to get the illusion of altitude?

13. The use of tools in geography (like the sextant, the heliotrope invented by Gauss, etc) to measure the Earth. How to measure the height of a mountain? How to draw maps of a region?

14. What is the shortest path on the Earth between two points: geodesics are given by the great circles.

15. The Earth is best modeled by an ellipsoid of revolution called a geoid. Explanation why the Earth has a larger radius at the equator. Geodesics on the geoid. They may not be in a plane.

16. Geometric grids on the Earth to make numerical computations. When making computations on the surface of the Earth, it is natural to divide the surface in small surface elements. If these are determined by small increases in longitude and latitude, then there are singularities at the poles. Geodesic grids could be more convenient for such calculations. They are linked to polyhedra inscribed in the sphere.

17. Movement of tectonic plates, continental drifts, rifts. Mathematics studies the dynamics of the planet mantle as an application to geosciences.

A planet supporting life

18. The phylogenetic tree of life. In biology, phylogenetics is the study of evolutionary relatedness among species. A phylogenetic tree or evolutionary tree is a branching diagram or tree showing the inferred evolutionary relationships among various biological species or other entities based upon similarities and differences in their physical and/or genetic characteristics. Computational phylogenetics is concerned by applying algorithms to assemble a phylogenetic tree representing a hypothesis about the evolutionary ancestry of a set of genes or species. It is a tool for taxonomy which is the science of classifying organisms.

19. Population models. Models of epidemics. Invasive species.

20. The chemical basis of morphogenesis: a simple reaction-diffusion model can explain the embryogenesis (the differentiation of cells starting from the spherical embryo), the pattern of animal coats, patterns on sea shells, phyllotaxis, etc.

A planet organized by humans

21. Transport systems: how to organize transport systems in efficient ways. Examples: modeling a transport network by a graph; optimization of a transport network; the traveling salesman problem; the Hamiltonian path problems; the postal system.

22. The web graph: it is a way of connecting the planet together.
Random walk on the web graph and the functioning of Google's PageRank algorithm?

23. Management of resources.

24. Game theory and applications in economics and in biology. Nash equilibria.

25. The economy of solidarity and how to fight poverty around the world: for instance the micro-credit.

A planet at risk

26. Climate models: how to use chaotic weather previsions where no prevision is valid past 14 days to long term climate previsions. One technique is to consider an average of many simulations with close initial conditions.

27. Hydrographic previsions: what will be the quantity of rain expected in a given region? This is important for agriculture, but also for filling the reservoirs of electric dams.

28. Evolution of the human population over history. Previsions for the next centuries.

29. Percolation: at each occupied node of a vertex there is a probability p that an adjacent node becomes occupied. Depending on p, the final configuration can have very different properties. Percolation models are useful to study the diffusion of liquids including pollutants inside soils. They also provide models for the diffusion of epidemics.

30. The age of the Earth. The first serious attempts to compute it were done by Lord Kelvin around 1840. Kelvin used Fourier's law of heat, with the gradient of temperature measured empirically and some very strong hypotheses simplifying the problem. He gave an interval of 24 to 400 million years. This estimate was in contradiction with the observations of the geologists and it was incompatible with the new theory of evolution of Darwin, which required a much older planet. Kelvin has neglected to take into accounts the convection movements inside the Earth, which slow down considerably the cooling of the mantle and also radioactivity, which is a source of heat.

31. The rising of the sea level with the melting of the glaciers: computation of the approximate rising of the sea level if all glaciers in Greenland and Antarctica were to melt.

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